Book of abstracts

A Gamma-convergence result and an application to the derivation of the Monge-Ampère gravitational model

Luigi Ambrosio Wed 9:00

Abstract: I will present a general Gamma-convergence result for action functionals involving a kinetic term plus a term depending on the gradient of a possibly nonsmooth convex function. The motivation for this problem came from the formal derivation of the Monge-Ampere gravitational model as a limit of partictle system. This is joint work with A.Baradat and Y.Brenier.

In the end I will also illustrate a generalization to metric spaces, proved in a joint work with C.Brena.

Persistence in functional topology and data analysis

Ulrich Bauer Tue 11:15

Abstract. Topological persistence is a cornerstone of applied topology, enabling the use of methods from algebraic topology in novel and powerful techniques of data analysis. In my talk, I will illustrate the central role and the historical development of persistent homology in different incarnations, connecting recent developments in topological data analysis with classical results in critical point theory and the calculus of variations. Presenting recent work with M. Schmahl and A. Medina-Mardones [1], I will explain how modern persistence theory provides a new and clarifying perspective on Morse's theory of functional topology, which has been instrumental in the first proof of the existence of unstable minimal surfaces by Morse and Tompkins.

[1] Bauer, U., Medina-Mardones, A. M., & Schmahl, M. (2021). Persistence in functional topology and a correction to a theorem of Morse. arXiv preprint arXiv:2107.14247.

Convergence under Kato bound for the Ricci curvature

Gilles Carron Fri 11:15

Abstract. I will present new results about limits of manifolds with a Kato bound on the negative part of the Ricci tensor. In the non-collapsing case, we recover part of the regularity theory that was known in the setting of Ricci lower bounds: in particular, we obtain that all tangent cones are metric cones, a stratification result and volume convergence to the Hausdorff measure.

It is a joint work with I. Mondello (Paris XII) and D. Tewodrose (U.L. Bruxelles).

Flatness results for stable solutions to some nonlocal problems

Eleonora Cinti Thu 14:00

Abstract. We study stable solutions to two, closely related, nonlocal problems: the fractional minimal surfaces equation and the fractional Allen-Cahn equation. In particular, we establish optimal energy estimates, whose local analogue is still unknown. These estimates, together with other ingredients (such as density estimates and a monotonicity formula) allow to prove flatness of level sets.

These results have been obtained in collaboration with X. Cabré, J. Serra, and E. Valdinoci.

Dirichlet spaces with distribution-valued Ricci bounds

Matthias Erbar Mon 14:50

Abstract: In this talk I will present a theory of tamed spaces which are Dirichlet spaces with distribution- valued lower bounds on the Ricci curvature seen from an Eulerian point of view. The approach is based on the analysis of singular perturbations of Dirichlet forms by a broad class of distributions. The distributional Ricci bound is then formulated in terms of an integrated version of the Bochner inequality generalizing the well-known Bakry-Emery curvature-dimension condition. Among other things we show the equivalence of distributional Ricci bounds to gradient estimates for the heat semigroup in terms of the Feynman-Kac semigroup induced by the taming distribution as well as consequences in terms of functional inequalities. I will many examples of tamed spaces including in particular Riemannian manifolds with either interior singularities or singular boundary behavior.

This is joint work with Chiara Rigoni, Theo Sturm, and Luca Tamanini.

On recent progress on the optimal matching problem

Michael Goldman Tue 14:00

Abstract: The optimal matching problem is a classical random combinatorial problem which may be interpreted as an optimal transport problem between random measures. Recent years have seen a renewed interest for this problem thanks to the PDE ansatz proposed in the physics literature by Caracciolo and al. and partially rigorously justified by Ambrosio-Stra-Trevisan. In this talk I will show how this ansatz combined with subadditivity may be used to give information both on the optimal cost and on the structure of the optimal transport map at various scales.

This is based on joint works with L. Ambrosio, M. Huesmann, F. Otto and D. Trevisan.

Discrete curvatures: flows and regularity

Felix Günther Tue 9:00

Abstract. In this talk we give an overview of ongoing work on several aspects of discrete curvatures. We start with discrete mean curvature flows of polygons and polyhedra, also known as flows by crystalline curvature. We sketch the known behaviour of the discrete curve shortening flow and give a (new) proof that the area-preserving discrete curve shortening flow lets convex curves converge to the anisotropic unit circle. After indicating obstacles that arise in the case of polyhedra, we move on to a theory of discrete mean and Gaussian curvature on polyhedral surfaces that makes the crystalline curvatures more local. It turns out that these discrete curvatures give rise to a notion of regularity of polyhedral surfaces that is closely related to the previously established notion of smoothness of polyhedral surfaces.

The talk is based on joint works with Alexander Bobenko, Le Nam Quang Do, Jan Techter and Christian Müller, Helmut Pottmann.

Tensor surface finite elements

Hanne Hardering Thu 11:15

Abstract: When discretizing a tangential tensor field equation on a curved smooth surface using a finite element approach, one is faced with three decisions: How to discretize the surface, how to approximate the tangentiality, and how to approximate the underlying function space. Each decision may lead to a non-conformity in the approximation, to failures of continuity, and to approximation errors. We will discuss this using the tensor-valued Laplace equation as a model and concentrating mainly on surface finite elements as discretization method.

Glued spaces and Ricci curvature bounds

Christian Ketterer Mon 16:00

Abstract. I will review some classical theorems about glued spaces and lower curvature bounds. Then I will present a recent result together with Vitali Kapovitch and Karl-Theodor Sturm showing that in the class of Alexandrov spaces equipped with a semi-concave weight and natural boundary conditions the Riemannian curvature-dimension condition (RCD) is preserved under gluing constructions with optimal lower curvature bounds.

Whitney manifold germs aka Manifolds with wild bounday

Peter Michor Wed 10:20

Abstract: During the preparation of a foundational chapter on manifolds of mappings for a book on geometric continuum mechanics I found out that the following object behaves surprisingly well as source of a manold of mappings:
— A Whitney manifold germ $\tilde{M} \supset M$ consists of an open manifold $\tilde{M}$ together with a closed subset $M \subset \tilde{M}$ which is the closure of its open interior, such that there exists a continuous linear extension operator from the space of Whitney jets on M to the space of smooth functions $C^\infty(\tilde{M})$, with their natural locally convex topologies. This concept is local in $\tilde{M}$, due to recent advances for the existence of continuous Whitney extension operators by D. Vogt, M. Tidten, L. Frerick, and J. Wengenroth. This notion is more general than all existing notions: domains with Lipschitz boundary or Hölder boundary, the manifolds with rough boundary of Roberts and Schmeding.
— The following concepts are very well behaved: Smooth mappings into manifolds. Vector bundles. Fiber bundles. The space of vector fields on M tangent to the boundary is convenient Lie algebra, with "Lie group" (in a weakened sense) the group of diffeomorphisms of M.
— Many open problems remain, in particular the Theorem of Stokes and the behaviour of the Dirichlet problem.

Optimal Transport, weak Laplacian bounds and minimal boundaries in non-smooth spaces with Lower Ricci Curvature bounds

Andrea Mondino Mon 14:00

Abstract. The goal of the seminar is to report on recent joint work with Daniele Semola, motivated by a question of Gromov to establish a "synthetic regularity theory" for minimal surfaces in non-smooth ambient spaces. In the setting of non-smooth spaces with lower Ricci Curvature bounds: - We establish a new principle relating lower Ricci Curvature bounds to the preservation of Laplacian bounds under the evolution via the Hopf-Lax semigroup; - We develop an intrinsic viscosity theory of Laplacian bounds and prove equivalence with other weak notions of Laplacian bounds; - We prove sharp Laplacian bounds on the distance function from a set (locally) minimizing the perimeter: this corresponds to vanishing mean curvature in the smooth setting; - We study the regularity of boundaries of sets (locally) minimizing the perimeter, obtaining sharp bounds on the Hausdorff co-dimension of the singular set plus content estimates and topological regularity of the regular set. Optimal transport plays the role of underlying technical tool for addressing various points.

Branched Covering Surfaces - New Shapes, New Materials and New Processes

Konrad Polthier Tue 14:50

Abstract: The classic geometric view on smooth surfaces hardly fits to the complex and often multiscale physical surface shapes in nature and, nowadays, in industrial applications. In this talk we will introduce a new class of multi-layered surface shapes derived from recent algorithms in geometry processing and related to classic complex analysis. Multivalued functions and differential forms naturally lead to the concept of branched covering surfaces and more generally of branched covering manifolds in the spirit of Hermann Weyl's book "The Idea of

a Riemann Surface " from 1913. We will illustrate and discretize basic concepts of branched (simplicial) covering surfaces starting from complex analysis and surface theory up to their recent appearance in geometry processing algorithms and artistic mathematical designs. Applications will touch discrete and differential surface modeling, image and geometry retargeting, optimal surfaces, and novel weaved geometry representations.

Stochastic mean curvature flow for graphs

Matthias Röger Tue 10:20

Abstract. We consider the evolution of graphs under a stochastically perturbed mean curvature flow. The noise term is white in time and possibly colored in space. We show suitable a priori estimates, prove existence of solutions in arbitrary dimensions and characterize the long-time behavior in the case of spatially homogeneous noise.

This is joint work with Nils Dabrock and Martina Hofmanova (Universität Bielefeld)

Compact manifolds with Kato-bounded Ricci curvature

Christian Rose Fri 9:00

Abstract. It is a classical fact that all compact Riemannian manifolds with prescribed uniform lower bound on the Ricci curvature and upper diameter bound possess similar geometric and spectral estimates. In the last decades there was an increasing interest in relaxing the uniform lower Ricci curvature bounds to integral bounds as they are

more stable with respect to perturbations of the metric. Even more general than the usual L^p-curvature restrictions is the so-called Kato condition on the negative part of the Ricci curvature. It proved to be a powerful tool to generalize many of the existing estimates on eigenvalues, heat kernel, and Betti number estimates, which I will discuss in my talk. In parts joint work with Gilles Carron and Guofang Wei.

Splines in the space of images

Martin Rumpf Thu 9:00

Abstract: This talk discusses variational models for spline curves in the space of images. The major focus is on such curves in the Riemannian image metamorphosis model and a comparison to splines is Wasserstein spaces. The task is to compute a smooth interpolation of key frames in the space of images. The original metamorphosis model is based on a simultaneous transport of image intensities and a modulation of intensities along motion trajectories and the energy functional measures the motion velocity and the material derivative of the image intensity. As in the case of cubic splines in Euclidean space where cubic splines are known to minimize the squared acceleration along the interpolation path we consider different acceleration terms to define a spline metamorphis model. In fact, the proposed spline functional combines quadratic functionals of the Eulerian motion acceleration and of the second material derivative representing an acceleration in the change of intensities along motion paths. Furthermore, a variational time discretization of this spline model is proposed and the convergence to a suitably relaxed time continuous model is discussed via Γ-convergence.

Variational Convergence of Liquid Crystal

Energies to Line and Surface Energies

Dominik Stantejsky Thu 14:50

Abstract. In this talk I will present a variational convergence result originating in the Landau-de Gennes model for nematic liquid crystals describing the Saturn ring effect around a spherical particle with external magnetic field. I will describe some properties of the limit functional and explain how this result can be generalized to non-spherical and in particular non-convex particles.

Curve Based Approximation of Measures on Manifolds by Discrepancy Minimization

Gabriele Steidl Thu 10:20

Abstract. The approximation of probability measures on compact metric spaces and in particular on Riemannian manifolds by atomic or empirical ones is a classical task in approximation and complexity theory with a wide range of applications. Instead of point measures we are concerned with the approximation by measures supported on Lipschitz curves. Special attention is paid to push-forward measures of Lebesgue measures on the interval by such curves. Using the discrepancy as distance between measures, we prove optimal approximation rates in terms of Lipschitz constants of curves. Having established the theoretical convergence rates, we are interested in the numerical minimization of the discrepancy between a given probability measure and the set of push-forward measures of Lebesgue measures on the interval by Lipschitz curves. We present numerical examples for measures on the 2- and 3-dimensional torus, the 2-sphere, the rotation group and the Grassmannian of all 2-dimensional linear subspaces in

4-dimensional Euclidean space. Our algorithm of choice is a conjugate gradient method on these manifolds which incorporates second-oder information. For efficiently computing the gradients and the Hessians within the algorithm, we approximate the given measures by truncated Fourier series and use fast Fourier transform techniques on these manifolds.

Joint work with M. Ehler, M. Gräf and S. Neumayer

Conformally Invariant Random Fields, Quantum Liouville Measures, and Random Paneitz Operators on Riemannian Manifolds of Even Dimension

Karl-Theodor Sturm Thu 16:00

Abstract. For large classes of even-dimensional Riemannian manifolds (M, g), we construct and analyze conformally invariant random fields. These centered Gaussian fields $h = h_g$, called *co-polyharmonic Gaussian fields*, are characterized by their covariance kernels k which exhibit a precise logarithmic divergence: $\left|k(x,y) - \log \frac{1}{d(x,y)}\right| \leq C$. They share a fundamental quasi-invariance property under conformal transformations. In terms of the co-polyharmonic Gaussian field h, we define the *quantum Liouville measure*, a random measure on M, heuristically given as

$$d\mu_g^h(x) := e^{\gamma h(x) - \frac{\gamma^2}{2} k(x,x)} \, d\,\mathsf{vol}_g(x)\,,$$

and rigorously obtained a.s. as weak limit of the RHS with h replaced by suitable regular approximations $h_\ell, \ell \in \mathbb{N}$. In terms on the quantum Liouville measure, we define the *Liouville Brownian motion* on M and the *random GJMS operators*.

Finally, we present an approach to a conformal field theory in arbitrary even dimensions with an ansatz based on Branson's Q-curvature: we give a rigorous meaning to the *Polyakov–Liouville measure*

$$d\boldsymbol{\nu}_g^*(h) = \frac{1}{Z_g^*} \exp\left(-\int \Theta\, Q_g h + m e^{\gamma h} d\, \mathsf{vol}_g\right) \exp\left(-\frac{a_n}{2} \mathfrak{p}_g(h,h)\right) dh$$

and we derive the correspondign conformal anomaly.

The class of admissible manifolds is conformally invariant. It includes all compact 2-dimensional Riemannian manifolds, all compact non-negatively curved Einstein manifolds of even dimension, and large classes of compact hpyervbolic manifolds of even dimension. However, not every compact even-dimensional Riemannian manifold is admissible.

Our results rely on new sharp estimates for heat kernels and higher order Green kernels on arbitrary compact manifolds.

Joint work with Lorenzo Dello Schiavo, Ronan Herry, Eva Kopfer

Breaking the curse of dimension in smooth optimal transport

François-Xavier Vialard Tue 16:00

Abstract: We show how to break the curse of dimension for the estimation of optimal transport distance between two smooth distributions for the Euclidean squared distance. The approach relies on essentially one tool: represent inequality constraints in the dual formulation of OT by equality constraints with a sum of squares in reproducing kernel Hilbert space. By showing this representation is tight in the variational formulation, one can then leverage smoothness to break the curse.