Analytic Methods in Complex Geometry 2023
Title and Abstracts

August 4, 2023

Ethan Addison (Stony Brook)

A Flowing Construction to Generalize Poincaré-Type Metrics

Poincar-type metrics are a flavor of cusp metrics on the complement of a divisor in a compact Kähler manifold exhibiting many friendly geometric properties. Yet, as shown by H. Auvray in the context of Calabi's extremal metrics, they have certain limitations owing to their sensitivity to the geometry of the ends. We introduce a construction called *gnarling* which augments Poincar-type metrics by incorporating certain holomorphic flows along the divisor. After outlining several aspects of these gnarled metrics, including a key growth estimate, we show their utility in perturbing classes of cscK Poincar-type metrics.

Fatima Zahra Assila (Chouabi Doukkali)

Projective Logarithmic Potentials and Their Applications

We study the projective logarithmic potential $\mathbb{G}_\mu$ of a probability measure μ on the complex projective space $\mathbb{P}^n$. We prove that the Green operator $\mathbb{G} : \mu \to \mathbb{G}_\mu$ has strong regularizing properties.

We show that the range of the operator $\mathbb{G}$ is contained in the (local) domain of definition of the complex Monge-Ampère operator on $\mathbb{P}^n$.

We show that the complex Monge-Ampère measure $(\omega + dd^c\mathbb{G}_\mu)^n$ of the logarithmic potential of μ is absolutely continuous with respect to the Lebesgue measure on $\mathbb{P}^n$ if and only if the measure μ has no atoms. Further, we give some applications of projective logarithmic potentials. First we introduce the notions of projective logarithmic energy and capacity associated to projective kernel. We compare quantitatively the projective logarithmic capacity with the complex Monge-Ampère capacity on $\mathbb{P}^n$ and we deduce that the set of zero logarithmic capacity is of Monge-Ampère capacity zero.

Shih-Kai Chiu (Oxford)

Calabi-Yau manifolds with maximal volume growth

Calabi-Yau manifolds with maximal volume growth are complete, non-compact Ricci-flat Kähler manifolds such that the volume grows at a rate of a cone towards infinity. These manifolds arise as desingularizations of affine varieties and as bubbles in the singularity formation of compact Kähler-Einstein manifolds. The maximal volume growth condition generalizes the more well-known notion of asymptotically conical (AC) manifolds. In this talk, I will survey recent development including new examples and uniqueness results.

Ronan Conlon (UT Dallas)

Two-dimensional shrinking Kahler-Ricci solitons

Shrinking Kahler-Ricci solitons model finite-time singularities of the Kahler-Ricci flow, hence the need for their classification. I will talk about the classification of such solitons in 4 real dimensions. This is joint work with Deruelle-Sun, Cifarelli-Deruelle, and Bamler-Cifarelli-Deruelle.

Tamas Darvas (U. Maryland)

Transcendental Okounkov bodies

We show that the volume of transcendental big (1,1)-classes on compact Kahler manifolds can be realized by convex bodies, thus answering questions of Lazarsfeld–Mustata and Deng. In our approach we use an approximation process by partial Okounkov bodies, and we study the extension of Kahler currents. (Joint work with R. Reboulet, M. Xia, D. Witt Nystrom, K. Zhang)

Slawomir Dinew (Jagiellonian U.)

Viscosity solutions and removal of singularities in complex geometry

We shall review recent developments in viscosity theory associated to nonlinear elliptic operators on complex manifolds. We shall emphasize the role of removability of singularities.

Henri Guenancia (UPS, Toulouse)

Strict positivity for Kähler-Einstein currents

I will report on a recent joint work with V. Guedj and A. Zeriahi. Given a compact Kähler space with klt singularity X, we study when a singular Kähler-Einstein metric on X dominates a Kähler form. I will discuss partial results when X has smoothable isolated singularities, as well as an unconditional result in dimension three.

Eveline Legendre (Lyon)

TBA TBA

Claude LeBrun (Stony Brook)

Gravitational Instantons, Weyl Curvature, and Conformally Kaehler Geometry

This talk will describe my recent joint work with Olivier Biquard and Paul Gauduchon on ALF Ricci-flat Hermitian 4-manifolds that are not hyper-Kaehler. Our main result essentially characterizes the known solutions by means of an open, purely Riemannian curvature condition.

Yang Li (MIT)

Recent progress on metric SYZ conjecture

Based on a number of motivations from differential geometry and mirror symmetry, Strominger-Yau-Zaslow proposed in the 90s that for Calabi-Yau manifolds near the large complex structure limit, there is a special Lagrangian torus fibration. I will try to survey the history on the metric aspect of this conjecture, outline what is expected in complex 3D and beyond, what has been proved, and what we still do not know. Time permitting, I will give a little more details on the example of Fermat family.

Hongyi Liu (UC Berkeley)

A compactness theorem for hyperkähler 4-manifolds with boundary

A hyperKähler 4-manifold is a Ricci-flat Kähler manifold with a global nowhere-vanishing holomorphic volume form. Forgetting about complex structures, it can be described purely in terms of a triple of symplectic 2-forms that are pointwise orthonormal with respect to the wedge product. In this talk, we consider compact hyperKähler 4-manifolds with boundary. Given a compact smooth 4-manifold with boundary and imposing some topological assumptions, we showed that if the boundary value of a sequence of hyperKähler triples converges smoothly to a non-degenerate triple of ”positive mean curvature”, then the triples themselves also converge to a smooth limiting hyperKähler triple. We prove this by obtaining a suitable Cheeger-Gromov compactness result. Since we do not focus much on complex structures, our method can be generalized to Einstein 4-manifolds with boundary.

Heather Macbeth (Fordham)

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Nick McCleerey (Michigan)

TBA TBA

Annamaria Ortu (SISSA)

A moduli space of holomorphic submersions

Proper holomorphic submersions of Kähler manifolds can be thought of as both a generalisation of holomorphic vector bundles and as a way of studying the behaviour of Kähler manifolds in families. On holomorphic submersions, we will define a stability condition for the fibres in terms of K-stability and we will describe a generalisation of a Hermite-Einstein connection, called an optimal symplectic connection. Using this condition we will discuss a deformation theory for fibrations that admit an optimal symplectic connection and we will construct their moduli space.

Tristan Ozuch (MIT)

Holonomy of limits of Einstein 4-manifolds

Recent developments have led to a complete reconstruction of the moduli space of Einstein 4-metrics in a Gromov-Hausdorff (GH) neighborhood of any noncollapsed singular metric. This arbitrarily precise reconstruction has provided indications that being Kähler may be a necessary condition for an Einstein singular space to be a limit of smooth Einstein 4-manifolds.

However, with Claude Lebrun, we show that this condition is not sufficient when the scalar curvature is positive. Specifically, our work demonstrates that if a real Einstein 4-metric is close to a Kähler Einstein orbifold with positive scalar curvature, it must be conformally Kähler, which rules out many desingularizations. To prove this, we combine the above reconstruction and a flexible criterion developed by Wu and improved by Lebrun to detect Kähler-Einstein metrics among Einstein metrics.

Mihai Paun (Bayreuth)

Construction of HE metrics in singular settings

We will report on the joint work arXiv:2303.08773. One of the main points of the paper is the use of a very flexible mean-value inequality obtained recently by [GPS] (Guo, Phong and Sturm) in the construction of HE metrics for stable coherent sheaves on compact Kähler spaces with reasonable singularities.

Duong Phong (Columbia)

The Monge-Ampère equation in Kähler geometry

In this colloquium-style talk, we discuss how the Monge-Ampère equation arises in Kähler geometry, and motivate the search for sharp geometric estimates for it. A survey is then given of recent advances using auxiliary equations, including joint works of the speaker with B. Guo and F. Tong on C^0 estimates, and with B. Guo, J. Song, and J. Sturm on the Green?s function and applications to the Kähler-Ricci flow.

Sisi Shen (Columbia)

Some canonical almost-Kähler metrics

We briefly discuss the literature on the existence of canonical metrics in the almost-Kähler setting. Based on a construction of Aazami and Ream, we show how to construct extremal and second-Chern Einstein non-Kähler almost Kähler metrics dual to a class of Lorentzian metrics called general plant-front waves. This is joint work with Mehdi Lejmi.

Marcin Sroka (Jagiellonian)

Fully nonlinear PDEs motivated by quaternionic geometry

More than a decade ago Alesker and Verbitsky introduced the analogue of Calabi's (volume prescribing) conjecture for hypercomplex manifolds. This problem also reduces to solvability of certain second order PDE called the quaternionic Monge-Ampere equation. Despite the conjecture remains open, recent years witnessed some partial results. I will outline these advances as well as discuss on more general PDEs motivated by quaternionic geometry and relations of the latter with already studied real and complex PDEs.

Ioana Suvaina (Vanderbilt)

Asymptotically Locally Euclidean Kähler Manifolds

The study of asymptotically locally Euclidean Kähler manifolds has seen a tremendous amount of development in the last few years. This talk will survey the main results and open problems in the area. The theory of ALE Ricci-flat Kähler manifolds is now well understood. The case of ALE scalar-flat Kähler manifolds is by contrast the focus of a great deal of current research. However, one can currently give a systematic account of which complex structures are possible; they are obtained by smoothings and resolutions of isolated quotient singularities. We will describe some nice corollaries of this fact in complex dimension two.

Freid Tong (Harvard)

Free boundary problem for Monge-Ampère equations and complete Calabi-Yau metrics

I will discuss a new free boundary problem for a real Monge-Ampere equation with applications to the construction of complete Calabi-Yau metrics. This is based on ongoing joint work with Tristan Collins and S.-T. Yau.

Christina Tonnesen-Friedman (Union College)

Extremal Sasaki metrics on Sphere Bundles

The purpose of the talk is to discuss the Sasakian geometry on odd dimensional sphere bundles arising from adapting the so-called fiber join construction for K-contact manifolds, introduced by T. Yamazaki, around the turn of the century.

One of the ongoing goals is to achieve a better understanding of the existence and non-existence of extremal and constant scalar curvature Sasaki metrics on such manifolds.

The talk is based primarily on past and ongoing work with Charles P. Boyer, Eveline Legendre, and Hongnian Huang.

Qi Yao (Münster)

Geodesic Equations on Asymptotically locally Euclidean Kähler manifolds

In this talk, we consider the geodesic equation in the space of Kähler metrics under the setting of ALE Kähler manifolds. The full $\mathcal{C}^{1,1}$ regularity of the solution will be proved. Then, we relate the solution of the geodesic equation to the uniqueness of scalar-flat ALE metrics. To this end, we study the asymptotic behavior of ε-geodesics at spatial infinity. Under the assumption that the Ricci curvature of a reference ALE Kähler metric is non-positive, we can prove the convexity of Mabuchi K-energy along ε-geodesics. However, by testing the Ricci curvature of ALE Kähler metrics, we find that on the family of negative line bundles $\mathcal{O}(-k)$ over $\mathbb{CP}^{n-1}$ with $n \geq 2$ and $k \neq n$, all ALE Kähler metrics cannot have non-positive (or non-negative) Ricci curvature.